

【解】(a)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 4 & 2 & 0 & 1 & 0 \\ 4 & 2 & 26 & 0 & 0 & 1 \end{array} \right] \begin{array}{l} \xrightarrow{(-4)} \\ \leftarrow \end{array} \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 4 & 2 & 0 & 1 & 0 \\ 0 & 2 & 10 & -4 & 0 & 1 \end{array} \right] \begin{array}{l} \xrightarrow{(-1/2)} \\ \leftarrow \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 4 & 2 & 0 & 1 & 0 \\ 0 & 0 & 9 & -4 & -1/2 & 1 \end{array} \right] \sim \dots$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 25/9 & 2/9 & -4/9 \\ 0 & 1 & 0 & 2/9 & 5/18 & -1/18 \\ 0 & 0 & 1 & -4/9 & -1/18 & 1/9 \end{array} \right]$$

$$\therefore A^{-1} = \begin{bmatrix} 25/9 & 2/9 & -4/9 \\ 2/9 & 5/18 & -1/18 \\ -4/9 & -1/18 & 1/9 \end{bmatrix}$$

(b) 由前面的列運算得知:

$$\therefore A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 4 & 2 \\ 0 & 0 & 9 \end{bmatrix} \quad (\text{LU分解: 綜線CH3定理27})$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 4 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}
\end{aligned}$$

(c) 如(b), U 為三角矩陣且主對角線元素皆正.

$\det U = U$ 的主對角線元素乘積 $\neq 0$,

$\therefore U$ 可逆.

(綜線CH4定理17)

$\therefore \ker U = \{o\}$.

(綜線CH8定理17)

$\therefore X \neq O \implies UX \neq O$.

$\therefore X^T A X = X^T U^T U X = (UX)^T (UX) > 0$

(綜線CH9範例1a)

4. (6% \times 3=18%) 機率