國防管理學院87年國防資訊研究所

1 (9%) 【國防87資訊】

Let $\begin{bmatrix} 1 & -1 & 3 \\ -1 & 2 & -3 \\ 3 & -3 & x^2 \end{bmatrix}$ be the augmented matrix of a system of linear equations over R.

Determine all values of x such that

(1) the system has no solution.

(2) the system has infinitely many solutions.

(3) the system has a unique solution.

【分析】本題屬於題型03B. 引用自清大72計管[7].

【解】(細節略)

(1) $x = -3$, (2) $x = 3$, (3) $x \neq \pm 3$

2.(11%) 【國防87資訊】

Find A^{-1}, where $A= \begin{bmatrix} -1 & 1 & 16 & 2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 6 \\ 0 & 1 & 1 & -3 \end{bmatrix}$

【分析】本題屬於題型03D. 引用自清大69計管[1]

【解】(細節略)

$$\begin{bmatrix} -1 & 85/2 & -55/2 & 1 \\ 0 & -9/2 & 7/2 & 1 \\ 0 & 3 & -2 & 0 \\ 0 & -1/2 & 1/2 & 0 \end{bmatrix}$$

3 (10%) 【國防87資訊】

Prove that there exists a linear transformation T:$R^2 \longrightarrow R^3$ such that $T(1, 1)=(1, 0, 2)$ and $T(2, 3)=(1, -1, 4)$, what is $T(8, 11)$=?

【分析】本題屬於題型07A. 請參閱綜線CH7範例4.

【解】(細節略)

$T(8, 11)=(5, -3, 16)$

4. (12%) 【國防87資訊】

Let T : $R^4 \longrightarrow R^3$ be the linear transformation defined by

$$T(x, y, z, w)=(x-2y+z-w, 3x-2z+3w, 5x-4y+w)$$

(1) Fine basis for the image of $T^{-1}(\mathrm{Im}(T))$. (6%)

(2) Find the value of a if $(1,4,a)\in \mathrm{Im}(T)$. (6%)

【分析】本題屬於題型06C. 修改自淸大75計管[2].

【解】(1)

設 $A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & 0 & -4 \\ 1 & -2 & 0 \\ -1 & 3 & 1 \end{bmatrix}$, 則 $T(x, y, z, w)=(x, y, z, w)A$

A經列運算化為 $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

$\mathrm{Im}(T)=\mathrm{RSP}(A)=\{s_1(1, 0, 2)+s_2(0, 1, 1) \mid s_1, s_2\in \mathbb{R}\}$

解 $(x, y, z, w)A=s_1(1, 0, 2)+s_2(0, 1, 1)$,

即 $A^{\mathrm{T}}(x, y, z, w)^{\mathrm{T}}=s_1(1, 0, 2)^{\mathrm{T}}+s_2(0, 1, 1)^{\mathrm{T}}$

$$\left[\begin{array}{cccc|cc} 1 & -2 & 1 & -1 & 1 & 0 \\ 3 & 0 & -2 & 3 & 0 & 1 \\ 5 & -4 & 0 & 1 & 2 & 1 \end{array}\right] \overset{\text{列運算}}{\sim \cdots \sim} \left[\begin{array}{cccc|cc} 1 & 0 & -2/3 & 1 & 0 & 1/3 \\ 0 & 1 & -5/6 & 1 & -1/2 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

此式即

$x+0y-(2/3)z+w=(1/3)s_2$,

$0x+y-(5/6)z+w=(-1/2)s_1+(1/6)s_2$.

$\therefore\ x=(2/3)t_1-t_2+(1/3)s_2$, $y=(5/6)t_1-t_2-(1/2)s_1+(1/6)s_2$, $z=t_1$, $w=t_2$,

$\therefore\ T^{-1}(\mathrm{Im}T)$

$=\{((2/3)t_1-t_2+(1/3)s_2 , (5/6)t_1-t_2-(1/2)s_1+(1/6)s_2, t_1, t_2) \mid s_1, s_2, t_1, t_2\in \mathbb{R}\}$

$=\{t_1(2/3, 5/6, 1, 0)+t_2(-1, -1, 0, 1)+s_1(0, -1/2, 0, 0)+s_2(1/3, 1/6, 0, 0)$

$\mid s_1, s_2, t_1, t_2\in \mathbb{R}\}$

$=\{k_1(4, 5, 6, 0)+k_2(1, 1, 0, 1)+k_3(0, 1, 0, 0)+k_4(2, 1, 0, 0)\mid k_1, k_2, k_3, k_4\in \mathbb{R}\}$

(2) (細節略) $a=6$.

5 (16%) 【國防87資訊】

Diagonalize the matrix A to PDP^{-1} and find P and D where

$$A=\begin{bmatrix}-1 & 4 & -2\\ -3 & 4 & 0\\ -3 & 1 & 3\end{bmatrix}$$

【分析】本題爲題型12C-08. 引用自中央85資工[6]

【解】(細節略)

$$P=\begin{bmatrix}1 & 2 & 1\\ 1 & 3 & 3\\ 1 & 3 & 4\end{bmatrix},\quad D=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$$

6 (12%) 【國防87資訊】

(1) Describe the Cayley-Hamilton theorem. (6%)

(2) Let $A=\begin{bmatrix}1 & -1 & 4\\ 3 & 2 & -1\\ 2 & 1 & -1\end{bmatrix}$ whose characteristic polynomial is $-\lambda^3+2\lambda^2+5\lambda-6$, Use the Cayley-Hamilton theorem to compute $A^6-2A^5-5A^4+3A^3+6A^2+15A-18I$, where I is the identity matrix. (6%)

【分析】本題爲題型16C-10之前半部. 引用自中正80資工[1]

【解】(細節略) 所求爲$O_{3\times 3}$.

7 (15%) 【國防87資訊】

Find the least squares approximating quardratic $y=a+bx+cx^2$ for the following five data points (–3, 3), (–1, 1), (0, 1), (1, 2), (3, 4)

【分析】本題爲題型09E-11. 引用自淡江84資工[3].

【解】(細節略)

$y=(121/105)+(1/5)x+(11/42)x^2$.

8 (15%) 【國防87資訊】

Let $A=\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Write $A=LU$, where L is a lower triangular matrix and U is an upper triangular matrix.

【分析】本題屬於題型03E. 引用自台大74資工甲乙[5]

【解】(細節略)

$$L=\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix},\quad U=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}.$$