淡江大學87資工所

1. 2. (離散數學)

3. (22%)　【淡江87資工】

Let P_2 be the set of all polynomial functions of degree ≤2. Consider P_2 with the inner product $\langle p,q\rangle = \int_{-1}^{1} p(x)q(x)dx$. Use the Gram-Schmidt process to transform the given basis $\{u_1, u_2, u_3\}$ into an orthogonal basis, where

$u_1=1+x^2$, $u_2=1-x^2$, $u_3=x$.

【分析】本題屬於題型09C.

【解】$v_1=u_1=1+x^2$, $\langle v_1, v_1\rangle = 56/15$, $\langle u_2, v_1\rangle = 8/5$

$v_2=u_2-(\langle u_2, v_1\rangle/\langle v_1, v_1\rangle)v_1 = (1-x^2) - (3/7)(1+x^2) = (2/7)(2-5x^2)$.

$\langle v_2, v_2\rangle=8/21$, $\langle u_3, v_1\rangle=0$, $\langle u_3, v_2\rangle=0$,

$v_3=u_3-(\langle u_3, v_1\rangle/\langle v_1, v_1\rangle)v_1-(\langle u_3, v_2\rangle/\langle v_2, v_2\rangle)v_2 = u_3 - 0v_1 - 0v_2 = x$.

$\langle v_3, v_3\rangle=2/3$.

所求之ortjhonormal basis為 $(15/56)^{1/2}(1+x^2)$, $(21/8)^{1/2}(2-5x^2)$, $(3/2)^{1/2}x$

4.(28%)　【淡江87資工】

Let $A=\begin{bmatrix} 5 & 3 & -7 \\ -1 & 1 & 1 \\ 3 & 3 & -5 \end{bmatrix}$

(a). Find the eigenvalues of A.

(b). Find a basis for each eigenspace of A.

(c). Find a matrix P that diagonalizes A and determine $P^{-1}AP$.

(d). Use diagonalization to compute A^{10}.

【分析】本題(a)(b)(c)屬於題型12C. 本題(d)屬於題型16B.

【解】(細節略)

(a) $\det(A-xI)=-x^3+x^2+4x-4=-(x-1)(x-2)(x+2)$. A的eigenvalue爲1,2,–2.

(b) 1,2,–2的eigenspace的基底分別取爲 $\{[1,1,1]^T.\}$, $\{[1,-1,0]^T.\}$, $\{[1,0,1]^T.\}$

(c)

$$P=\begin{bmatrix}1&1&1\\1&-1&0\\1&0&1\end{bmatrix},\quad D=P^{-1}AP=\begin{bmatrix}1&0&0\\0&2&0\\0&0&-2\end{bmatrix}$$

(d) $A^{10}=(PD^{10}P^{-1})=P\text{diag}(1,1024,1024)P^{-1}$

$$=\begin{bmatrix}1&-1023&1023\\-1023&1&1023\\-1023&-1023&2047\end{bmatrix}$$