交通大學87工業工程所

1 (10%) 【交大87工工】

$$A=\begin{bmatrix} 2 & 0 & 0 \\ 0 & \sqrt{2} & -\sqrt{2} \\ 0 & \sqrt{2} & \sqrt{2} \end{bmatrix};\quad A^{-1}=?$$

【分析】本題屬於題型03D.

【解】依標準做法可輕易解得

$$A^{-1}=\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & \sqrt{2}/4 & \sqrt{2}/4 \\ 0 & -\sqrt{2}/4 & \sqrt{2}/4 \end{bmatrix}$$

【另解】(此法稍快, 但公式不熟的同學避免使用)

令$B=(1/2)A$, 觀察得知B的各行形成正交單位集,

所以$B^{-1}=B^{T}$. (CH2定義25, CH13定理3)

$A^{-1}=(2B)^{-1}=(1/2)B^{-1}=(1/2)B^{T}=(1/2)((1/2)A)^{T}=(1/4)A^{T}=$(答案同上)

2 (10%) 【交大87工工】

Let D be the differential operator on P_3, Find the matrix A representing D with respect to $[1, 2x, 4x^2-1]$, an ordered basis in P_3. P_3 is the set of all polynomials of degree less than 3

【分析】本題屬於題型07B.

【解】$D(1) = 0 = 0\cdot 1 + 0\cdot 2x + 0\cdot(4x^2-1)$,

$D(2x) = 2 = 2\cdot 1 + 0\cdot 2x + 0\cdot(4x^2-1)$

$D(4x^2-1) = 8x = 0\cdot 1 + 4\cdot 2x + 0\cdot(4x^2-1)$

$\therefore\ A=\begin{bmatrix}0&2&0\\0&0&4\\0&0&0\end{bmatrix}$　　(綜線CH7定義9)

3 (10%)　【交大87工工】

Let A be a 10×9 matrix and the rank of A is 7.

(1) What is the number of independent constraints for $AX=O$　(4分)

(2) What is the dimension of $C=\{X \mid AX=O\}$?　(3分)

(3) What is the number of redundant constraints for $AX=O$　(3分)

【分析】本題屬於題型03B.

constraint(未知數的限制條件)並非通用的正式術語, 只是有些書試圖用它來講解維度概念. 考生遇到這種名詞就用常理推測其含意.

【解】(2) $\dim C = 9-\text{rank}A$　　(綜線CH8定理8)

$=2$

(1) 由矩陣乘法得知方程式$AX=O$的X是9×1矩陣.

X內含9個未知數, 解出之C爲$\mathbb{R}^9$中的2維子空間,

$\therefore$ independent constraints共有7個.

(3) 將$AX=O$乘開後共有10個等式, 扣除7個independent constraint,

$\therefore$ 內含3個redundant constraint.

4 (10%)　【交大87工工】

Let $U_1=\begin{bmatrix}3\\2\end{bmatrix}$; $U_2=\begin{bmatrix}1\\1\end{bmatrix}$; and $X=\begin{bmatrix}5\\4\end{bmatrix}$. Find the coordinates of X with respect to the ordered basis $[U_1, U_2]$.

【分析】本題屬於題型06D.

【解】

設 $a\begin{bmatrix}3\\2\end{bmatrix}+b\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}5\\4\end{bmatrix}$.

比較分量並解方程式得 $a=1, b=2$.

∴ 所求爲 $\begin{bmatrix}1\\2\end{bmatrix}$.