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參考章節使用簡稱, 例如綜線CH3代表廖亦德著:「綜合線性代數」第3章.
題型代表廖亦德著:「線性代數題型剖析」書中的題型.
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科目: 計算機數學 (線代50%, 離散50%)

1. (10%) 【成大88資工】

Explain the following terms.

(a) Rank equation

(b) singular matrix

(c) Row echelon form

(d) Homogeneous system

【分析】本題屬於題型17E.

【解】(a) 此名詞來歷不明, 有需要的同學請到圖書館搜尋.

(b) 可逆矩陣. 見綜線CH2定義10.

(c) 列簡化梯形矩陣, 見綜線CH3定義4.

(d) 齊次方程組, 見綜線CH3定義1.

2. (10%) 【成大88資工】

Let u=[–1, 2] and v=[3, –5] be in $\mathbb{R}^2$ (Euclidean 2-space), and let T: $\mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that $T(u)$=[–2, 1, 0] and T(v)=[5, –7, 1]. Find the standard matrix representation A of T and compute T([–4, 3]).

【分析】本題屬於題型07A. 請參閱綜線CH7範例4及CH定義9.

【解】 (細節略)

先將[x y]表成u, v的線性組合，再依線性條件即可解得

$T([x, y]) = [-y, -9x-4y, 2x+y]$

T對標準基底的矩陣表示爲 $\begin{bmatrix} 0 & -1 \\ -9 & -4 \\ 2 & 1 \end{bmatrix}$ (CH7定理9a)

$T([-4, 3]) = [-3, 24, -5]$

3. (10%) 【成大88資工】

Find the least-squares solution of the given overdetermined system $Ax=b$ by converting it to a consistent system and then solving.

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

【分析】本題屬於題型09E. 請參閱綜線CH9範例21b.

【解】(細節略)

normal equation $A^{T}Ax=A^{T}b$爲

$$\begin{bmatrix} 14 & 3 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

其解爲 $x_1=-1/5$, $x_2=3/5$.

4. (10%) 【成大88資工】

Find the eigenvalues λ_i and the corresponding eigenvectors v_i of the given matrix A, and also find an invertible matrix C and a diagonal matrix D such that $D=C^{-1}AC$.

$$A=\begin{bmatrix} 6 & 3 & -3 \\ -2 & -1 & 2 \\ 16 & 8 & -7 \end{bmatrix}$$

【分析】本題屬於題型12C. 請參閱綜線CH12範例17.

【解】(細節略)

∴ eigenvalues爲 1, 0, –3.

eigenvalue 1 的eigenvector 爲 $k[\,0 \quad 1 \quad 1\,]^T$, $k\neq 0$.

eigenvalue 0 的eigenvector 爲 $k[\,-1 \quad 2 \quad 0\,]^T$, $k\neq 0$.

eigenvalue –3 的eigenvector 爲 $k[\,1 \quad -1 \quad 2\,]^T$, $k\neq 0$.

令$C=\begin{bmatrix} 0 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$, $D=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

則 $D=C^{-1}AC$

5. (10%) 【成大88資工】

Find an orthonormal basis for a subspace in $\mathbb{R}^4$ being spanned by v_1, v_2 and v_3, i.e., $W = \mathrm{sp}(v_1, v_2, v_3)$, if $v_1 = [1, 1, 1, 1]$, $v_2 = [-1, 1, -1, 1]$ and $v_3 = [1, -1, -1, 1]$. Then find the projection of $b=[1, 2, 3, 4]$ on W.

【分析】本題屬於題型09B. 請參閱綜線CH9定理12.

【解】本題由內積可知 v_1, v_2, v_3 兩兩正交，且長度都是2,

∴ orthonormal basis可取爲

$\{[\,1/2 \quad 1/2 \quad 1/2 \quad 1/2\,], [\,-1/2 \quad 1/2 \quad -1/2 \quad 1/2\,], [\,1/2 \quad -1/2 \quad -1/2 \quad 1/2\,]\}$

所求之projection爲

$$\frac{bv_1^T}{v_1v_1^T}v_1+\frac{bv_2^T}{v_2v_2^T}v_2+\frac{bv_3^T}{v_3v_3^T}v_3=(5/2)v_1+(1/2)v_2+0v_2=[\,2 \quad 3 \quad 2 \quad 3\,]$$