台灣師範大學88資教所

科目: 線性代數與離散數學

1. -- 8 離散數學考題 (共100%)

9. (16%) 【師大88資教】

(a) From $AB=C$, find a formula for A^{-1} .

(b) From $PA=LU$, find a formula for A^{-1} .

【分析】本題屬於題型02A.

本題(a)應添加“A, C 皆可逆”的條件. 本題(b)應添加“A, LU 皆可逆”的條件.

【解】(a) 原式兩邊左乘A^{-1}得 $B=A^{-1}C$, 兩邊再右乘C^{-1}得$A^{-1}=BC^{-1}$.

(b) 原式兩邊右乘A^{-1}得$P=LUA^{-1}$, 兩邊再左乘$(LU)^{-1}$得$(LU)^{-1}P=A^{-1}$.

$\therefore$ $A^{-1}=(LU)^{-1}P$, 若L, U都可逆, 則$A^{-1}=U^{-1}L^{-1}P$.

10. (22%) 【師大88資教】

Find the least squares solution to the system

$$\begin{cases} x_1 + x_2 = 3 \\ -2x_1 + 3x_2 = 1 \\ 2x_1 - x_2 = 2 \end{cases}$$

【分析】本題屬於題型09E. 請參閱綜線CH9定理21a

【解】(細節略)

原方程式為 $\begin{bmatrix} 1 & 1 \\ -2 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

原方程式為 $\begin{bmatrix} 9 & -7 \\ -7 & 11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

解得x_1=83/50, x_2=71/50

11. (28%) 【師大88資教】

Find the limiting values of y_k and z_k ($k \to \infty$) if

$y_{k+1} = 0.8\, y_k + 0.3\, z_k$, $y_0 = 0$

$z_{k+1} = 0.2\, y_k + 0.7\, z_k$, $z_0 = 5$

Also find formula for y_k and z_k from $A^k = S\Lambda^k S^{-1}$, where $A= \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$ and it can be diagonalized as $A = S\Lambda S^{-1}$.

【分析】本題屬於題型16E. 請參閱綜線CH16範例14.

【解】(細節略)

對A作對角化得$A = S\Lambda S^{-1}$, 其中

$S= \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$, $\Lambda= \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} y_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 0.8\, y_k + 0.3\, z_k \\ 0.2\, y_k + 0.7\, z_k \end{bmatrix} = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} y_k \\ z_k \end{bmatrix} = A \begin{bmatrix} y_k \\ z_k \end{bmatrix}$$

$$\begin{bmatrix} y_k \\ z_k \end{bmatrix} = A^k \begin{bmatrix} y_0 \\ z_0 \end{bmatrix} = S\Lambda^k S^{-1} \begin{bmatrix} y_0 \\ z_0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0.5^n & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} -3\,(0.5)^n+3 \\ 3\,(0.5)^n+2 \end{bmatrix}$$

∴ $y_k = -3\,(0.5)^n + 3$, $z_k = 3\,(0.5)^n + 2$

極限值爲 $\lim y_k = 3$, $\lim z_k = 2$.

12. (34%) 【師大88資教】

Suppose we want to minimize $x_1 - x_2$ subject to

$2x_1 - 4x_2 + x_3 = 6$

$3x_1 + 6x_2 + x_4 = 12$

where all $x_i \geq 0$. Starting from X=(0, 0, 6, 12),

(a) should x_1 or x_2 be increased from its current value of zero ?

(b) how far can it be increased until the equations forces x_3 or x_4 down to zero ?

(c) at that point what is X ?

【分析】本題超出線性代數範圍. 請參閱線性規劃的相關書籍.

【解】(略)