

交通大學88統計所

科目: 微積分與線性代數

1. (10%)

Let M be an $n \times n$ matrix. If the sum of the entries in each row of M is 1, show that 1 is an eigenvalue of M .

【分析】本題屬於題型12A. 本題很簡單, 但沒見過的人很難想出解法.

【解】設 $u = [1, 1, \dots, 1]^T$, 則 $Mu = [1, 1, \dots, 1]^T = u$,
 $\therefore u$ 為 M 的 eigenvector, 以 1 為 eigenvalue.

2. (20%)

For an $n \times n$ matrix $M = (m_{ij})_{n \times n}$, $\text{trace}(M)$ is defined as $m_{11} + \dots + m_{nn}$.

(a) Let M_1 and M_2 be $n_1 \times n_2$ and $n_2 \times n_1$ matrices, respectively. Show that

$$\text{trace}(M_1 M_2) = \text{trace}(M_2 M_1) \quad (10\%)$$

(b) Let M and S be $n \times n$ matrices. Show that

$$\text{trace}(S^{-1} M S) = \text{trace}(M) \quad (10\%)$$

【分析】本題屬於題型02C. 請參閱綜線CH2定理28.

【解】請參閱綜線CH2定理28的證明, 此處不再重複.

3. (20%)

An $n \times n$ matrix M is said to be symmetric if $M = M^T$, where M^T is the transpose of M . An $n \times n$ symmetric matrix M is said to be positive definite if $x^T M x$ is positive for any $n \times 1$ column vector x , where x^T is the transpose of x . Let M be an $n \times n$ symmetric matrix.

(a) If M is positive definite, show that all the eigenvalues of M are positive. (10%)

(b) If all the eigenvalues of M are positive, show that M is positive definite. (10%)

【勘誤】 題目中positive matrix的定義 “for any $n \times 1$ column vector x ”, 應改爲
“for any $n \times 1$ non-zero column vector x ”.

【分析】 本題屬於題型13D. 請參閱綜線CH13定理17c, CH13定理15.
本題的矩陣都必須是實數矩陣.

【解】 請參閱綜線CH13定理17c的證明. 此處不再重複.

4. (20%), 5. (10%), 6. (10%), 7. (10%), [微積分]
