線性代數解析—中正89資工所 廖亦德 解

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[1]. (10%) 【中正89資工】

Detrmine true or false for the following problems:

(a) If matrix A has repeated eigenvalues, then A is not diagonalized.

(b) If matrix A has different eigenvalues, then A is invertible.

(c) If V and W are both subspaces of a given vector space, the $(V\cap W)$ and $(V\cup W)$ are still subspaces, but $(V+W)$ is not a subspace.

(d) The mapping $T(x_1, x_2)=(x_1+x_2-1, x_1-x_2)$ is not a linear mapping.

(e) The dimension of $W=\{(x_1, x_2, x_3, x_4, x_5, x_6)|\ x_1+x_3+x_5=0,\ x_2+x_4+x_6=0\ \}$ is equal to 2.

【勘誤】本題(a)的diagonalized應改為diagonalizable.

【分析】本題(a)屬於題型12A. 參閱綜線CH12定理21

本題(b)屬於題型12A. 參閱綜線CH14定理2b, CH12定理23

本題(c)屬於題型05B. 參閱綜線CH5定理22,24,27.

本題(d)屬於題型07A. 參閱綜線CH7範例7.

本題(d)屬於題型08E. 參閱綜線CH8範例9.

【解】(a) False, (b) False, (c) False, (d) True, (e) False.

【說明】(a) 反例如$A=I_3$, 其eigenvalues為1,1,1. 但仍可對角化. (已對角化)

(b) 反例如$A=\text{diag}(2, 0)$, 其eigenvalue為2,0. 但不可逆.

(c) 應該是: $(V\cap W)$ and $(V+W)$ are still subspaces, (綜線CH5定理22, CH5定理27)

but $(V\cup W)$ is not a subspace. (綜線CH5範例22a, CH5定理24)

(d) $T(0,0)$不等於$(0,0)$.

(e) 令$T(x_1, x_2, x_3, x_4, x_5, x_6)=(x_1+x_3+x_5, x_2+x_4+x_6)$, 則

$W=\text{Ker}T$,

$\dim \text{Ker}T = 6-\dim \text{Im}(T)$ (綜線CH8定理8)

$= 6 - \dim R^2 = 4$

[2]. (10%) 【中正89資工】

Let $A\in R^{n\times n}$. As we know, A is invertible(non-singular) if and only if the linear system $AX=O$ has only trivial solution. Find other five equivalent conditions which are equivalent to "A is invertible "

【分析】本題屬於題型08E. 請參閱綜線CH8定理17.

【解】本題請參閱綜線CH8定理17自行挑選性質作答.

[3]. (10%) 【中正89資工】

Let $A=\begin{bmatrix} 1 & -1 & 1 \\ 4 & 0 & -1 \\ 4 & -2 & 1 \end{bmatrix}$

(a) Diagonal A.

(b) Find $\det(mA^{2m}-nA^{2n})$ where $m,n\in N$.

【分析】本題(a)屬於題型12C. 請參閱綜線CH12範例17.

本題(b)屬於題型16B. 請參閱綜線CH16定理2.

【解】(a) $\det(A-xI)=...=-(x-2)(x-1)(x+1)$

解$(A-2I)v=o$, 得2的特徵向量$[1, 1, 2]^T$,

解$(A-I)v=o$, 得1的特徵向量$[1, 2, 2]^T$,

解$(A+I)v=o$, 得-1的特徵向量$[0, 1, 1]^T$,

令$P=\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix}$, 則$P^{-1}AP=\text{diag}(2,1,-1)$

(b) $\det(mA^{2m}-nA^{2n})$

$=\det(m(P\text{diag}((2,1,-1))P^{-1})^{2m}-n(P\text{diag}((2,1,-1))P^{-1})^{2n})$

$=\det(P(m\,\text{diag}(2,1,-1)^{2m}-n\,\text{diag}((2,1,-1)^{2n})P^{-1})$ (綜線CH16定理2)

$=\det(m\,\text{diag}(2,1,-1)^{2m}-n\,\text{diag}((2,1,-1)^{2n})$ (綜線CH7定理22)

$=\det(m\,\text{diag}(2^{2m},1^{2m},(-1)^{2m})-n\,\text{diag}((2^{2n},1^{2n},(-1)^{2n}))$ (綜線CH2定理4b)

$=\det(m\,\text{diag}(2^{2m},1,1)-n\,\text{diag}((2^{2n},1,1))$

$=\det\text{diag}(m2^{2m}-n2^{2n},m-n,m-n)$

$=(m2^{2m}-n2^{2n})(m-n)^2$ (綜線CH4定理4a)

[4]. (10%) 【中正89資工】

Let $T: V\to W$ be a linear transformation. Show that T is one-to-one $\Longleftrightarrow$ $\ker(T)=\{o\}$ where $\ker(T)=\{x\in V \mid T(x)=o\}$.

【分析】本題屬於題型08A. 請參閱綜線CH8定理7.

【解】本題請參閱綜線CH8定理7, 此處不再重複.

[5]. (10%) 【中正89資工】

(a) Show that $\{1, \cos x, \cos 2x, ..., \cos nx\}$ is an orthogonal set.

(b) Find an orthonormal set based on the orthogonal set stated in (a).

【分析】本題屬於題型09A. 請參閱綜線CH9範例6.

【解】此題為綜線CH9範例6的一部份, 請自行參閱作答.