線性代數解析--成大89資工所

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1. (10%) 【成大89資工】

Check if the following matrices are diagonalizable. If Diagonalizable, compute a matrix that diagonalizes the given matrix.

(a) $\begin{bmatrix} 8 & -7 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} -7 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ -4 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

【分析】本題屬於題型12B及12C. 請參閱綜線CH12範例12.

【解】(a) $\det(A-xI)=x^2(x-8)(x-1)$

$\dim(\ker(A-0I))=4-\text{rank}(A-0I)=4-3=1$,

此geometric multiplicity<2

∴ 不可對角化. (綜線CH12定理21)

(b) $\det(A-xI)=...=x(x-1)(x-\alpha)(x-\beta)$,

其中 $\alpha, \beta = (-5\pm\sqrt{65})/2$

此題有四個相異特徵值, ∴可對角化.

分別由 $(A-0I)v=o$, $(A-1I)v=o$, $(A-\alpha I)v=o$, $(A-\beta I)v=o$,

各解出一個eigenvector, 再排成方陣即得. (請讀者自行求算)

2. (10%) 【成大89資工】

In the following, find the inverse of the matrix or else show that this matrix is singular.

(a) $\begin{bmatrix} -1 & 1 & 16 & 2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 6 \\ 0 & 1 & 1 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & 6 & 0 & 0 \\ 1 & 4 & 4 & 11 \\ 4 & -4 & -5 & 3 \\ -3 & 1 & 2 & -6 \end{bmatrix}$

【分析】本題屬於題型03D. 請參閱綜線CH4範例12b.

【解】(a) 以列運算求解:

$$\left[\begin{array}{cccc|cccc} -1 & 1 & 16 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 6 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -3 & 0 & 0 & 0 & 1 \end{array}\right] \sim \ldots \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & 85/2 & -55/2 & 1 \\ 0 & 1 & 0 & 0 & 0 & -9/2 & 7/2 & 1 \\ 0 & 0 & 1 & 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1/2 & 1/2 & 0 \end{array}\right]$$

所求為 $\begin{bmatrix} -1 & 85/2 & -55/2 & 1 \\ 0 & -9/2 & 7/2 & 1 \\ 0 & 3 & -2 & 0 \\ 0 & -1/2 & 1/2 & 0 \end{bmatrix}$

(b)

$$\left[\begin{array}{cccc|cccc} -2 & 6 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 4 & 4 & 11 & 0 & 1 & 0 & 0 \\ 4 & -4 & -5 & 3 & 0 & 0 & 1 & 0 \\ -3 & 1 & 2 & -6 & 0 & 0 & 0 & 1 \end{array}\right] \sim \ldots \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 3 & * & * & * & * \\ 0 & 1 & 0 & 1 & * & * & * & * \\ 0 & 0 & 1 & 1 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * & * & * \end{array}\right]$$

計算後發現原先的4×4矩陣rank為3, 所以不可逆. (CH8定理17)

3. (10%) 【成大89資工】

Find the general solution of the system or show that the system has no solution.

$$8x_2-4x_3+10x_6=1$$
$$x_3+x_5-x_6=2$$
$$x_4-3x_5+2x_6=0$$

【分析】本題屬於題型03A. 請參閱綜線CH3範例7.

【解】對分隔矩陣以列運算求解,

$$\left[\begin{array}{cccccc|c} 0 & 8 & -4 & 0 & 0 & 10 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 & -3 & 2 & 0 \end{array}\right] \sim \ldots \sim \left[\begin{array}{cccccc|c} 0 & 1 & 0 & 0 & 1/2 & 3/4 & 9/8 \\ 0 & 0 & 1 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 & -3 & 2 & 0 \end{array}\right]$$

通解爲 $x_1=t_1$,

$x_2=9/8-(1/2)t_5-(3/4)t_6=9/8$

$x_3=2-t_5+t_6$

$x_4=3t_5-2t_6$

$x_5=t_5$,

$x_6=t_6$, t_1, t_5, t_6爲任意參數.

4. (10%) 【成大89資工】

Show that A is row equivalent to B and B is row equivalent to C. Prove that A is row equivalent to C.

【分析】本題考基本定義及基本性質. 請參閱綜線CH3定義3.

【解】A is row equivalent to B 是說有一連串基本列運算可將A化爲B.

B is row equivalent to C 是說有一連串基本列運算可將B化爲C.

將這兩串基本列運算接起來就可以將A化爲C,

所以A is row equivalent to C.

5. (10%) 【成大89資工】

Determine the dimension of the subspace of R^3 consisting of all vectors parallel to a given plane through the origin. (Here, R^3 denotes a vector space with three components.)

【分析】本題屬於題型08E. 請參閱綜線CH8範例9.

【解】R^3中的平面可用方程式 $ax+by+cz=0$, (a, b, c不全爲零) 來表示.

令 $T: R^3 \to R^1$, 定義爲 $T(x, y, z)=ax+by+cz$, 則顯然T的值域爲ImT爲R.

$\dim\{ (x, y, z) \mid ax+by+cz=0 \}=\dim \{ (x, y, z) \mid T(x, y, z)=0 \}$

$= \dim(R^3)-\dim(\mathrm{Im}T)=3-1=2.$

【另解】也可確實求出基底, 但需討論a, b, c是否爲零, 反而比較麻煩.