# 人工智慧 （Artificial Intelligence）不確定知識和推理 

（Uncertain Knowledge and Reasoning）

1092AI05<br>MBA，IM，NTPU（M5010）（Spring 2021）<br>Wed 2，3， 4 （9：10－12：00）（B8F40）



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課程大綱（Syllabus）
週次（Week）日期（Date）内容（Subject／Topics）
1 2021／02／24 人工智慧概論
（Introduction to Artificial Intelligence）
2 2021／03／03 人工智慧和智慧代理人
（Artificial Intelligence and Intelligent Agents）
3 2021／03／10 問題解決
（Problem Solving）
4 2021／03／17 知識推理和知識表達
（Knowledge，Reasoning and Knowledge Representation）
5 2021／03／24 不確定知識和推理
（Uncertain Knowledge and Reasoning）
6 2021／03／31 人工智慧個案研究
（Case Study on Artificial Intelligence I）

## 課程大綱（Syllabus）

## 週次（Week）日期（Date）内容（Subject／Topics）

7 2021／04／07 放假一天（Day off）
8 2021／04／14 機器學習與監督式學習
（Machine Learning and Supervised Learning）
9 2021／04／21 期中報告
（Midterm Project Report）
10 2021／04／28 學習理論與綜合學習
（The Theory of Learning and Ensemble Learning）
11 2021／05／05 深度學習
（Deep Learning）
12 2021／05／12 人工智慧個案研究 II
（Case Study on Artificial Intelligence II）

## 課程大綱（Syllabus）

週次（Week）日期（Date）內容（Subject／Topics）
13 2021／05／19 強化學習
（Reinforcement Learning）
14 2021／05／26 深度學習自然語言處理
（Deep Learning for Natural Language Processing）
15 2021／06／02 機器人技術
（Robotics）
16 2021／06／09 人工智慧哲學與倫理，人工智慧的未來 （Philosophy and Ethics of AI，The Future of AI）
17 2021／06／16 期末報告 I
（Final Project Report I）
18 2021／06／23 期末報告 II
（Final Project Report II）

# Uncertain Knowledge and Reasoning 

## Outline

- Quantifying Uncertainty
- Probabilistic Reasoning
- Probabilistic Reasoning over Time
- Probabilistic Programming
- Making Simple Decisions
- Making Complex Decisions


## Stuart Russell and Peter Norvig (2020), <br> Artificial Intelligence: A Modern Approach,

4th Edition, Pearson


## Artificial Intelligence: A Modern Approach

1. Artificial Intelligence
2. Problem Solving
3. Knowledge and Reasoning
4. Uncertain Knowledge and Reasoning
5. Machine Learning
6. Communicating, Perceiving, and Acting
7. Philosophy and Ethics of AI

# Artificial Intelligence: 

## Uncertain

# Knowledge and Reasoning 

## Artificial Intelligence:

## 4. Uncertain Knowledge and Reasoning

- Quantifying Uncertainty
- Probabilistic Reasoning
- Probabilistic Reasoning over Time
- Probabilistic Programming
- Making Simple Decisions
- Making Complex Decisions
- Multiagent Decision Making


## Intelligent Agents

## 4 Approaches of AI

## 2.

Thinking Humanly:
The Cognitive
Modeling Approach
1.

Acting Humanly:
The Turing Test
Approach ${ }_{\text {(asso) }}$

## 3.

Thinking Rationally: The "Laws of Thought" Approach

## 4.

Acting Rationally:
The Rational Agent Approach

## Reinforcement Learning (DL)

## Agent

## Environment

## Reinforcement Learning (DL)



## Reinforcement Learning (DL)



## Agents interact with environments through sensors and actuators



# Quantifying Uncertainty 

## DT-Agent <br> A Decision-Theoretic Agent that Selects Rational Actions

function DT-AGENT( percept) returns an action
persistent: belief_state, probabilistic beliefs about the current state of the world action, the agent's action
update belief_state based on action and percept calculate outcome probabilities for actions,
given action descriptions and current belief_state
select action with highest expected utility
given probabilities of outcomes and utility information
return action

## Agent 1 has inconsistent beliefs

Proposition Agent 1's Agent 2 Agent1 Agent 1 payoffs for each outcome

$$
\begin{aligned}
& \text { belief bets bets } a, b a, \neg b \neg a, b \sim a, \neg b \\
& \begin{array}{llllllll}
a & 0.4 & \$ 4 o n a & \$ 60 n \sim a & -\$ 0 & -\$ 6 & \$ 4 & \$ 4
\end{array} \\
& \begin{array}{llllllll}
b & 0.3 & \$ 3 \text { on } b & \$ 70 n-b & -\$ 7 & \$ 3 & -\$ 7 & \$ 3
\end{array} \\
& \begin{array}{llllllll}
a \vee b & 0.8 & \$ 20 n-(a \vee b) & \$ 80 n a \vee b & \$ 2 & \$ 2 & \$ 2 & -\$ 8
\end{array} \\
& \begin{array}{llll}
-\$ 11 & -\$ 1 & -\$ 1 & -\$ 1
\end{array}
\end{aligned}
$$

## A full joint distribution for the Toothache, Cavity, Catch world

|  | tothache |  | Ttothache |  |
| :---: | :---: | :---: | :---: | :---: |
|  | atch | רatch | atch | fatch |
| canity | 0.108 | 0.012 | 0.072 | 0.008 |
| Cenaty | 0.016 | 0.064 | 0.14 | 0.576 |

## Weather and Dental problems are independent



## Coin flips are independent

## Coin $_{1} \ldots \ldots$ Coin $_{n}$

## decomposes into <br> 



# Probabilistic 

 Reasoning
## A Simple Bayesian Network

Weather is independent to the other three variables.
Toothache and Catch are conditionally independent, given Cavity.


## A Typical Bayesian Network

Topology and the Conditional Probability Tables (CPTs)


# Conditional Probability Table for P(Fever |Cold, Flu, Malaria) 

| Cold | Flu | Malaria $P($ fever $\mid \cdot) P(\neg$ fever $\mid \cdot)$ |  |  |
| :---: | :---: | :---: | :--- | :--- |
| $f$ | $f$ | $f$ | 0.0 | 1.0 |
| $f$ | $f$ | $t$ | 0.9 | $\mathbf{0 . 1}$ |
| $f$ | $t$ | $f$ | 0.8 | $\mathbf{0 . 2}$ |
| $f$ | $t$ | $t$ | 0.98 | $0.02=0.2 \times 0.1$ |
| $t$ | $f$ | $f$ | 0.4 | $\mathbf{0 . 6}$ |
| $t$ | $f$ | $t$ | 0.94 | $0.06=0.6 \times 0.1$ |
| $t$ | $t$ | $f$ | 0.88 | $0.12=0.6 \times 0.2$ |
| $t$ | $t$ | $t$ | 0.988 | $0.012=0.6 \times 0.2 \times 0.1$ |

## A Simple Network

with discrete variables (Subsidy and Buys) and continuous variables (Harvest and Cost )


## Probability distribution over Cost as a function of Harvest size


(a)
(b)
(c)
distribution P (Cost | Harvest ), obtained by summing over the two subsidy cases.

## A normal (Gaussian) distribution for the cost threshold


(a)

(b)

Expit and Probit models for the probability of buys given cost

## A Bayesian Network

## for evaluating car insurance applications



## The structure of the expression



## The Enumeration Algorithm

## for Exact Inference in Bayes Nets

function EnUMERATION- $\operatorname{ASK}(X, \mathbf{e}, b n)$ returns a distribution over $X$ inputs: $X$, the query variable
$\mathbf{e}$, observed values for variables $\mathbf{E}$
$b n$, a Bayes net with variables vars
$\mathbf{Q}(X) \leftarrow$ a distribution over $X$, initially empty
for each value $x_{i}$ of $X$ do
$\mathbf{Q}\left(x_{i}\right) \leftarrow$ Endmerate-All $\left(\right.$ vars, $\left.\mathbf{e}_{x_{i}}\right)$
where $\mathbf{e}_{x_{i}}$ is $\mathbf{e}$ extended with $X=x_{i}$
return Normalize $(\mathbf{Q}(X))$
function ENUMERATE-ALL(vars, e) returns a real number
if Empty? (vars) then return 1.0
$V \leftarrow$ FIRST(vars)
if $V$ is an evidence variable with value $v$ in $\mathbf{e}$ then return $P(v \mid$ parents $(V)) \times$ EnUMERAtE-AlL(Rest(vars), e) else return $\sum_{v} P(v \mid$ parents $(V)) \times$ Enumerate-All(Rest(vars), $\left.\mathbf{e}_{v}\right)$
where $\mathbf{e}_{v}$ is $\mathbf{e}$ extended with $V=v$

## Pointwise Multiplication $\mathrm{f}(X, Y) \times \mathrm{g}(\mathbb{Y}, Z)=\mathrm{h}(X, Y, Z)$

| $X$ | $Y$ | $\mathbf{f}(X, Y)$ | $Y$ | $Z$ | $\mathbf{g}(Y, Z)$ | $X$ | $Y$ | $Z$ | $\mathbf{h}(X, Y, Z)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t$ | $t$ | .3 | $t$ | $t$ | .2 | $t$ | $t$ | $t$ | $.3 \times .2=.06$ |
| $t$ | $f$ | .7 | $t$ | $f$ | .8 | $t$ | $t$ | $f$ | $.3 \times .8=.24$ |
| $f$ | $t$ | .9 | $f$ | $t$ | .6 | $t$ | $f$ | $t$ | $.7 \times .6=.42$ |
| $f$ | $f$ | .1 | $f$ | $f$ | .4 | $t$ | $f$ | $f$ | $.7 \times .4=.28$ |
|  |  |  |  |  |  | $f$ | $t$ | $t$ | $.9 \times .2=.18$ |
|  |  |  |  |  |  | $f$ | $t$ | $f$ | $.9 \times .8=.72$ |
|  |  |  |  |  |  | $f$ | $f$ | $t$ | $.1 \times .6=.06$ |
|  |  |  |  |  |  | $f$ | $f$ | $.1 \times .4=.04$ |  |

## The Variable Elimination Algorithm for Exact Inference in Bayes Nets

function Elimination- $\operatorname{AsK}(X, \mathbf{e}, b n)$ returns a distribution over $X$ inputs: $X$, the query variable
$\mathbf{e}$, observed values for variables $\mathbf{E}$
$b n$, a Bayesian network with variables vars
factors $\leftarrow[]$
for each $V$ in ORDER(vars) do
factors $\leftarrow[$ MAKE-FACTOR $(V, \mathbf{e})]+$ factors
if $V$ is a hidden variable then factors $\leftarrow$ Sum-OUT ( $V$, factors) return Normalize(POINTWISE-PRODUCT(factors))

## Bayes Net Encoding

of the 3-CNF (Conjunctive Normal Form) Sentence
$(W V X V Y) \wedge(\neg W V Y V Z) \wedge(X V Y \vee \neg Z)$


## Multiply Connected Network

(b) A clustered equivalent


## A Sampling Algorithm that generates events from a Bayesian network

function PRIOR-SAMPLE( $b n$ ) returns an event sampled from the prior specified by $b n$ inputs: bn, a Bayesian network specifying joint distribution $\mathbf{P}\left(X_{1}, \ldots, X_{n}\right)$
$\mathbf{x} \leftarrow$ an event with $n$ elements
for each variable $X_{i}$ in $X_{1}, \ldots, X_{n}$ do
$\mathbf{x}[i] \leftarrow \operatorname{arandom}$ sample from $\mathbf{P}\left(X_{i} \mid\right.$ parents $\left.\left(X_{i}\right)\right)$
return x

## The Rejection-Sampling Algorithm

 for answering queries given evidence in a Bayesian networkfunction Rejection-SAmpling $(X, \mathbf{e}, b n, N)$ returns an estimate of $\mathbf{P}(X \mid \mathbf{e})$
inputs: $X$, the query variable
$\mathbf{e}$, observed values for variables $\mathbf{E}$
$b n$, a Bayesian network
$N$, the total number of samples to be generated local variables: $\mathbf{C}$, a vector of counts for each value of $X$, initially zero
for $j=1$ to $N$ do
$\mathbf{x} \leftarrow$ PRIOR-SAMPLE $(b n)$
if $\mathbf{x}$ is consistent with $\mathbf{e}$ then
$\mathbf{C}[j] \leftarrow \mathbf{C}[j]+1$ where $x_{j}$ is the value of $X$ in $\mathbf{x}$ return Normalize( $\mathbf{C}$ )

## The Likelihood-Weighting Algorithm for inference in Bayesian networks

function Likelihood-Weighting $(X, \mathbf{e}, b n, N)$ returns an estimate of $\mathbf{P}(X \mid \mathbf{e})$
inputs: $X$, the query variable
$\mathbf{e}$, observed values for variables $\mathbf{E}$
$b n$, a Bayesian network specifying joint distribution $\mathbf{P}\left(X_{1}, \ldots, X_{n}\right)$
$N$, the total number of samples to be generated
local variables: $\mathbf{W}$, a vector of weighted counts for each value of $X$, initially zero
for $j=1$ to $N$ do
$\mathbf{x}, w \leftarrow$ Weighted-SAmple $(b n, \mathbf{e})$
$\mathbf{W}[j] \leftarrow \mathbf{W}[j]+w$ where $x_{j}$ is the value of $X$ in $\mathbf{x}$
return $\operatorname{NormALIZE}(\mathbf{W})$
function WEIGHTED-SAMPLE $(b n, \mathbf{e})$ returns an event and a weight
$w \leftarrow 1 ; \mathbf{x} \leftarrow$ an event with $n$ elements, with values fixed from $\mathbf{e}$
for $i=1$ to $n$ do
if $X_{i}$ is an evidence variable with value $x_{i j}$ in $\mathbf{e}$
then $w \leftarrow w \times P\left(X_{i}=x_{i j} \mid \operatorname{parents}\left(X_{i}\right)\right)$
else $\mathbf{x}[i] \leftarrow$ a random sample from $\mathbf{P}\left(X_{i} \mid\right.$ parents $\left.\left(X_{i}\right)\right)$
return $\mathbf{x}, w$

## Performance of rejection sampling and likelihood weighting on the insurance network



## The Gibbs Sampling Algorithm for approximate inference in Bayes nets

function $\operatorname{Gibbs}-\operatorname{Ask}(X, \mathbf{e}, b n, N)$ returns an estimate of $\mathbf{P}(X \mid \mathbf{e})$
local variables: $\mathbf{C}$, a vector of counts for each value of $X$, initially zero
$\mathbf{Z}$, the nonevidence variables in $b n$
$\mathbf{x}$, the current state of the network, initialized from $\mathbf{e}$
initialize $\mathbf{x}$ with random values for the variables in $\mathbf{Z}$
for $k=1$ to $N$ do
choose any variable $Z_{i}$ from $\mathbf{Z}$ according to any distribution $\rho(i)$ set the value of $Z_{i}$ in $\mathbf{x}$ by sampling from $\mathbf{P}\left(Z_{i} \mid m b\left(Z_{i}\right)\right)$
$\mathbf{C}[j] \leftarrow \mathbf{C}[j]+1$ where $x_{j}$ is the value of $X$ in $\mathbf{x}$
return Normalize(C)

## The States and Transition Probabilities

## of the Markov Chain

for the query $\mathbf{P}($ Rain $/$ Sprinkler $=$ true, WetGrass $=$ true $)$

(a)

(b)

Transition Probabilities when the CPT for Rain constrains it to have the same value as Cloudy

## Performance of Gibbs sampling compared to likelihood weighting on the car insurance network


(a)
for the standard query on PropertyCost

(b)
for the case where the output variables are observed and Age is the query variable

## A Causal Bayesian Network

 representing cause-effect relations among five variables
(a)

(b)

The network after performing the action "turn Sprinkler on."

# Probabilistic 

 Reasoning over Time
## Bayesian network structure

 corresponding to a First-order Markov Process with state defined by the variables $X t$.

# Bayesian Network Structure and Conditional Distributions describing the umbrella world 



## Smoothing computes $\mathrm{P}\left(\mathbf{X}_{\mathrm{k}} \mid \mathrm{e}_{1: \mathrm{t}}\right)$,

the posterior distribution of the state at some past time $k$ given a complete sequence of observations from 1 to $t$.


## The Forward-Backward Algorithm for Smoothing

function FORWARD-BACKWARD(ev, prior) returns a vector of probability distributions inputs: ev, a vector of evidence values for steps $1, \ldots, t$ prior, the prior distribution on the initial state, $\mathbf{P}\left(\mathbf{X}_{0}\right)$ local variables: fv, a vector of forward messages for steps $0, \ldots, t$
b, a representation of the backward message, initially all 1 s $\mathbf{s v}$, a vector of smoothed estimates for steps $1, \ldots, t$
$\mathbf{f v}[0] \leftarrow$ prior
for $i=1$ to $t$ do
$\mathbf{f v}[i] \leftarrow \operatorname{FORWARD}(\mathbf{f v}[i-1], \mathbf{e v}[i])$
for $i=t$ down to 1 do
$\mathbf{s v}[i] \leftarrow \operatorname{NORMALIZE}(\mathbf{f v}[i] \times \mathbf{b})$
$\mathbf{b} \leftarrow \operatorname{BACKWARD}(\mathbf{b}, \mathbf{e v}[i])$
return sv

## Possible state sequences for Rain

 can be viewed as paths through a graph of the possible states at each time step(a)
(b)


Operation of the Viterbi algorithm
for the umbrella observation sequence [true, true, false, true, true]

## Algorithm for Smoothing with a Fixed Time Lag of d Step

function Fixed-LAG-Smoothing $\left(e_{t}, h m m, d\right)$ returns a distribution over $\mathbf{X}_{t-d}$
inputs: $e_{t}$, the current evidence for time step $t$
$h m m$, a hidden Markov model with $S \times S$ transition matrix $\mathbf{T}$
$d$, the length of the lag for smoothing
persistent: $t$, the current time, initially 1
$\mathbf{f}$, the forward message $\mathbf{P}\left(X_{t} \mid e_{1: t}\right)$, initially $h m m$.PRIOR
$\mathbf{B}$, the $d$-step backward transformation matrix, initially the identity matrix
$e_{t-d: t}$, double-ended list of evidence from $t-d$ to $t$, initially empty
local variables: $\mathbf{O}_{t-d}, \mathbf{O}_{t}$, diagonal matrices containing the sensor model information
add $e_{t}$ to the end of $e_{t-d: t}$
$\mathbf{O}_{t} \leftarrow$ diagonal matrix containing $\mathbf{P}\left(e_{t} \mid X_{t}\right)$
if $t>d$ then
$\mathbf{f} \leftarrow \operatorname{FORWARD}\left(\mathbf{f}, e_{t-d}\right)$
remove $e_{t-d-1}$ from the beginning of $e_{t-d: t}$
$\mathbf{O}_{t-d} \leftarrow$ diagonal matrix containing $\mathbf{P}\left(e_{t-d} \mid X_{t-d}\right)$
$\mathbf{B} \leftarrow \mathbf{O}_{t-d}^{-1} \mathbf{T}^{-1} \mathbf{B T O}_{t}$
else $\mathrm{B} \leftarrow \mathrm{BTO}_{t}$
$t \leftarrow t+1$
if $t>d+1$ then return $\operatorname{Normalize}(\mathbf{f} \times \mathbf{B 1})$ else return null

## Specification of the prior, transition model, and sensor model for the umbrella DBN



## A DBN fragment

## the sensor status variable required for modeling persistent failure of the battery sensor


(a)

(b)

## Unrolling a

## Dynamic Bayesian Network



## The Particle Filtering Algorithm

function Particle-Filtering $(\mathbf{e}, N, d b n)$ returns a set of samples for the next time step inputs: e, the new incoming evidence
$N$, the number of samples to be maintained
$d b n$, a DBN defined by $\mathbf{P}\left(\mathbf{X}_{0}\right), \mathbf{P}\left(\mathbf{X}_{1} \mid \mathbf{X}_{0}\right)$, and $\mathbf{P}\left(\mathbf{E}_{1} \mid \mathbf{X}_{1}\right)$
persistent: $S$, a vector of samples of size $N$, initially generated from $\mathbf{P}\left(\mathbf{X}_{0}\right)$
local variables: $W$, a vector of weights of size $N$
for $i=1$ to $N$ do
$S[i] \leftarrow$ sample from $\mathbf{P}\left(\mathbf{X}_{1} \mid \mathbf{X}_{0}=S[i]\right) \quad / /$ step 1
$W[i] \leftarrow \mathbf{P}\left(\mathbf{e} \mid \mathbf{X}_{1}=S[i]\right) \quad / /$ step 2
$S \leftarrow$ Weighted-Sample-With-Replacement $(N, S, W) \quad / /$ step 3 return $S$

## The Particle Filtering Update Cycle for the Umbrella DBN


(a) Propagate


Rain $_{t+1}$

(b) Weight
(c) Resample

## A Dynamic Bayes Net

for simultaneous localization and mapping in the stochastic-dirt vacuum world


Source: Stuart Russell and Peter Norvig (2020), Artificial Intelligence: A Modern Approach, 4th Edition, Pearson

# Probabilistic <br> <br> Programming 

 <br> <br> Programming}

## Possible Worlds

for a language with two constant symbols, R and J


## Bayes Net for a Single customer C1 recommending a single book B 1 . Honest(C1) is Boolean



Bayes net with two customers and two books

## Bayes Net

for the book recommendation when Author(B2) is unknown


## One particular world for the book recommendation OUPM

| Variable | Value | Probability |
| :---: | :---: | :---: |
| \＃Customer | 2 | 0.3333 |
| \＃Book | 3 | 0.3333 |
| Honest ${ }_{\langle\text {Customer，，1＞}}$ | true | 0.99 |
| Honest ${ }_{\langle\text {Customer，，} 2\rangle}$ | false | 0.01 |
| Kindness $\langle$ Customer，，1〉 | 4 | 0.3 |
| Kindness $\langle$ Customer，，2〉 | 1 | 0.1 |
| Quality ${ }_{\text {SBook，，1＞}}$ | 1 | 0.05 |
| Quality SBook，，2＞$^{\text {，}}$ | 3 | 0.4 |
| Quality ${ }_{\text {SBook，，3＞}}$ | 5 | 0.15 |
| \＃LoginID ${ }_{\text {＜Owner，}\langle\text { Customer，，1＞＞}}$ | 1 | 1.0 |
| \＃LoginID $\langle$（Owner，$\langle$ Customer，，2〉＞ | 2 | 0.25 |
| Recommendation $\left\langle\right.$ LoginID，$\left\langle\right.$ Owner，$\langle\text { Customer，，1＞＞，1〉，} \text { Book }, 1\rangle^{\text {，}}$ | 2 | 0.5 |
| Recommendation $\langle$ LoginID，$\langle$ Owner，$\langle$ Customer，， 1$\rangle\rangle, 1\rangle,\langle$ Book，，2 $\rangle$ | 4 | 0.5 |
| Recommendation $\left\langle\right.$ LoginID，$\left\langle\right.$ Owner，$\langle\text { Customer，，1＞＞，1〉，} \text { Book }, ~, 3\rangle^{\text {，}}$ | 5 | 0.5 |
| Recommendation $\left\langle\right.$ LoginID，$\left\langle\right.$ Owner，$\langle\text { Customer，，2＞＞，1＞，} \text { Book }, ~, 1\rangle^{\text {，}}$ | 5 | 0.4 |
| Recommendation $\left\langle\right.$ LoginID，$\left\langle\right.$ Owner，$\langle\text { Customer，，2＞＞，1＞，} \text { Book }, ~, 2\rangle^{\text {，}}$ | 5 | 0.4 |
| Recommendation $\left\langle\right.$ LoginID，$\left\langle\right.$ Owner，$\langle\text { Customer，，2＞＞，1＞，} \text { Book }, ~, 3\rangle^{\text {，}}$ | 1 | 0.4 |
| Recommendation $\left\langle\right.$ LoginID，$\left\langle\right.$ Owner，$\langle\text { Customer，，2＞＞，2＞，} \text { Book }, ~, 1\rangle^{\text {，}}$ | 5 | 0.4 |
| Recommendation $\left\langle\right.$ LoginID，$\left\langle\right.$ Owner，$\langle\text { Customer，，2＞＞，2〉，} \text { Book，}, 2\rangle^{\text {，}}$ | 5 | 0.4 |
| Recommendation $\left\langle\right.$ LoginID，$\left\langle\right.$ Owner，$\langle\text { Customer，，2〉〉，2〉，} \text { Book，}, 3\rangle^{\text {，}}$ | 1 | 0.4 |

## An OUPM for

## Citation Information Extraction

```
type Researcher, Paper, Citation
random String Name(Researcher)
random String Title(Paper)
random Paper PubCited(Citation)
random String Text(Citation)
random Boolean Professor(Researcher)
origin Researcher Author(Paper)
#Researcher ~ OM (3,1)
Name(r) ~ NamePrior()
Professor(r) ~ Boolean(0.2)
#Paper (Author =r) ~ if Professor (r) then OM(1.5,0.5) else OM (1,0.5)
Title(p) ~ PaperTitlePrior()
CitedPaper(c) ~ UniformChoice({Paper p})
Text(c) ~ HMMGrammar(Name(Author(CitedPaper(c))),Title(CitedPaper(c)))
```


# Making Simple Decisions 

## Nontransitive preferences $\mathrm{A} \succ \mathrm{B} \succ \mathrm{C} \succ \mathrm{A}$

## can result in irrational behavior:

a cycle of exchanges each costing one cent

(a)

is equivalent to

(b)

The decomposability axiom

## The Utility of Money



## Unjustified optimism

 caused by choosing the best of $k$ options

## Strict dominance

## (a) Deterministic (b) Uncertain


(a)

(b)

## Stochastic dominance



(b)

Cumulative distributions for the frugality of S1 and S2.

## A decision network for the

airport-siting problem


# A simplified representation of the airport-siting problem 



# Making Complex Decisions 

## A dynamic decision network

for a mobile robot with state variables for battery level, charging status, location, and velocity, and action variables for the left and right wheel motors and for charging.


## The game of Tetris The DDN for the Tetris MDP



## The Value Iteration Algorithm for calculating utilities of states

function VALUE-ITERATION $(m d p, \epsilon)$ returns a utility function inputs: $m d p$, an MDP with states $S$, actions $A(s)$, transition model $P\left(s^{\prime} \mid s, a\right)$, rewards $R\left(s, a, s^{\prime}\right)$, discount $\gamma$
$\epsilon$, the maximum error allowed in the utility of any state
local variables: $U, U^{\prime}$, vectors of utilities for states in $S$, initially zero $\delta$, the maximum relative change in the utility of any state
repeat
$U \leftarrow U^{\prime} ; \delta \leftarrow 0$
for each state $s$ in $S$ do
$U^{\prime}[s] \leftarrow \max _{a \in A(s)} \quad$ Q-VALUE $(m d p, s, a, U)$
if $\left|U^{\prime}[s]-U[s]\right|>\delta$ then $\delta \leftarrow\left|U^{\prime}[s]-U[s]\right|$
until $\delta \leq \epsilon(1-\gamma) / \gamma$
return $U$

## AIMA Python

- Artificial Intelligence: A Modern Approach (AIMA)
- http://aima.cs.berkeley.edu/
- AIMA Python
- http://aima.cs.berkeley.edu/python/readme.html
- https://github.com/aimacode/aima-python
- Probability Models (DTAgent)
- http://aima.cs.berkeley.edu/python/probability.html
- Markov Decision Processes (MDP)
- http://aima.cs.berkeley.edu/python/mdp.html


# Python in Google Colab (Python101) 

https://colab.research.google.com/drive/1FEG6DnGvwfUbeo4zJ1zTunjMqf2RkCrT
co python101.ipynb - Colaborator $\times+$


- https://colab.research.google.com/drive/1FEG6DnGvwfUbeo4zJ1zTunjMqf2RkCrT?authuser=2\#scrollTo=wsh36fLxDKC3

```
*
```

$\triangle$ python101.ipynb
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CONNECTED

- EDITING
(1) 1 \# Future Value
$2 \mathrm{pv}=100$
$3 r=0$.
5 fv = pv * ((1 + (r)) ** n)
6 print(round(fv, 2))
$\zeta \quad 194.87$
[11] 1 amount $=100$
interest $=10 \# 10 \%=0.01 * 10$
years $=7$
5 future_value = amount * ((1 + (0.01 * interest)) ** years) 6 print(round(future_value, 2))
$\longmapsto \quad 194.87$
[12] 1 \# Python Function def
def getfv(pv, r, n):
$\mathrm{fv}=\mathrm{pv}$ * $((1+(\mathrm{r}))$ ** n$)$ return fv
$\mathrm{fv}=\operatorname{getfv}(100,0.1,7$.
print(round(fv, 2))
$\longmapsto \quad 194.87$
[13] 1 \# Python if else score $=80$
if score $>=60$
4 print("Pass")
5 else:
print(. "Fail".).
$\zeta$ Pass


## Summary

- Quantifying Uncertainty
- Probabilistic Reasoning
- Probabilistic Reasoning over Time
- Probabilistic Programming
- Making Simple Decisions
- Making Complex Decisions


## References

- Stuart Russell and Peter Norvig (2020), Artificial Intelligence: A Modern Approach, 4th Edition, Pearson.
- Aurélien Géron (2019), Hands-On Machine Learning with ScikitLearn, Keras, and TensorFlow: Concepts, Tools, and Techniques to Build Intelligent Systems, 2nd Edition, O’Reilly Media.

